International Journal of Mathematics
and Computer Applications Research (IJMCAR)
ISSN(P): 2249-6955; ISSN(E): 2249-8060
Vol. 5, Issue 5, Oct 2015, 1-6
TJPRC Pvt. Ltd.

STRONGLY MULTIPLICATIVE LABELING OF SOME PATH RELATED GRAPHS

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ABSTRACT

We discuss here Strongly Multiplicative labeling of some path related graphs. We prove that the Total Graph and Splitting Graph of the path P_n are strongly multiplicative. In addition to this we prove that Shadow Graph of the path and Triangular Snakes are strongly multiplicative.

KEYWORDS: Strongly Multiplicative Labeling; Total Graph; Splitting Graph; Shadow Graph

2010 Mathematics Subject Classification: 05C78.

INTRODUCTION

Throughout this work, by a graph we mean finite, connected, undirected, simple graph $G = (V(G), E(G))$ of order $|V(G)|$ and size $|E(G)|$.

Definition1.1: A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges) then the labeling is called a *vertex labeling (an edge labeling).*

A latest survey on various graph labeling problems can be found in Gallian [2].

Definition1.2: A graph $G = (V(G), E(G))$ with p vertices is said to be *multiplicative* if the vertices of G can be labeled with p distinct positive integers such that label induced on the edges by the product of labels of end vertices are all distinct.

Multiplicative labeling was introduced by Beineke and Hegde [1]. In the same paper they proved that every graph G admits multiplicative labeling and defined strongly multiplicative labeling as follows.

Definition1.3: A graph $G = (V(G), E(G))$ with p vertices is said to be strongly multiplicative if the vertices of G can be labeled with p consecutive positive integers $1,2,., p$ such label induced on the edges by the product of labels of end vertices are all distinct.

Beineke and Hegde [1] proved the following results.

- Every cycle C_n is strongly multiplicative.
- Every wheel W_n is strongly multiplicative.
- The complete graph K_n is strongly multiplicative $\Leftrightarrow$ n $\leq$ 5.
- The complete bipartite graph $K_{n,n}$ is strongly multiplicative $\Leftrightarrow$ n $\leq$ 4.

- Every spanning subgraph of a strongly multiplicative graph is strongly multiplicative.
- Every graph is an induced subgraph of a strongly multiplicative graph.

Vaidya and Kanani[5] have discussed strongly multiplicative labeling in the context of arbitrary supersubdivision of graphs and proved the following results.

- Arbitrary supersubdivision of the path P_n is strongly multiplicative.
- Arbitrary supersubdivision of the star $K_{1,n}$ is strongly multiplicative.
- Arbitrary supersubdivision of the cycle C_n is strongly multiplicative.

Here we consider the following definitions.

- The *Total Graph* $T(G)$ of G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent whenever they are either adjacent or incident in G.
- The *Splitting Graph* of the graph G is obtained by adding to each vertex v a new vertex v' such that v' is adjacent to every vertex which is adjacent to v in G in other words $N(v) = N(v')$. The Splitting graph is denoted by $S'(G)$.
- The *Shadow Graph* $D_2(G)$ of a connected graph G is constructed by taking two copies of G namely G' and G'' join each vertex u' in G' to the neighbors of the corresponding vertex u'' in G''.
- The *Triangular Snake* TS_n is obtained from the path P_n by replacing every edge of the path by the triangle C_3.

For any undefined term in graph theory we rely upon Gross and Yellen[3].

MAIN RESULTS

Theorem 2.1: The Total Graph $T(P_n)$ of the path P_n is strongly multiplicative.

Proof: Let $v_1, v_2, \ldots, v_n$ be the n vertices and $e_1, e_2, \ldots, e_{n-1}$ be the (n – 1) edges of the path P_n. Let $G = T(P_n)$ be the total graph of the path P_n. Let $v'_1, v'_2, \ldots, v'_{n-1}$ be the newly added vertices corresponding to edges $e_1, e_2, \ldots, e_{n-1}$ to obtain total graph of the path P_n. We note that $|V(G)| = 2n-1$ and $|E(G)| = 4n-5$.

We define strongly multiplicative labeling $f : V(G) \rightarrow \{1,2,3,\ldots,2n-1\}$ as follows.

$f(v_i) = 2i - 1; \qquad 1 \le i \le n;$

$f(v'_i) = 2i; \qquad 1 \le i \le n-1.$

The labeling pattern defined above covers all the possibilities and in each case the graph G under consideration admits strongly multiplicative labeling. That is, the Total Graph $T(P_n)$ of the path P_n is strongly multiplicative.

Illustration 2.2: The Total Graph $T(P_4)$ of path P_4 and its strongly multiplicative labeling is shown in *Figure 1*.

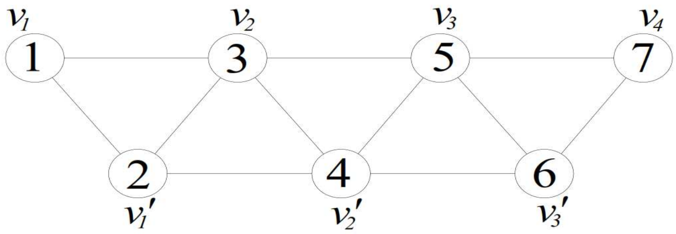

Figure 1: Strongly Multiplicative Labeling of Total Graph T(P_4).

Theorem 2.3: The Splitting Graph $S'(P_n)$ of the path P_n is strongly multiplicative.

Proof: Let P_n be the path with n-vertices and $v_1, v_2, \ldots\ldots, v_n$ be the vertices of path P_n. Let $G = S'(P_n)$ be the graph obtained by adding to each vertex v a new vertex v' such that v' is adjacent to every vertex that is adjacent to v in G. We note that $|V(G)| = 2n$ and $|E(G)| = 3n - 3$.

We define strongly multiplicative labeling $f : V(G) \rightarrow \{1,2,3,\ldots,2n\}$ as follows.

$f(v_i) = 2i; \quad 1 \leq i \leq n;$

$f(v'_i) = 2i-1; \quad 1 \leq i \leq n.$

The labeling pattern defined above covers all the possibilities and in each case the graph G under consideration admits strongly multiplicative labeling. That is, the Splitting Graph $S'(P_n)$ of the path P_n is strongly multiplicative.

Illustration 2.4: The Splitting Graph $S'(P_7)$ of path P_7 and its strongly multiplicative labeling is shown in *Figure 2*.

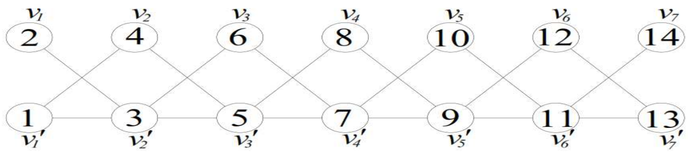

Figure 2 Strongly Multiplicative Labeling of Splitting Graph S′(P_7).

Theorem 2.5: The Shadow Graph $D_2(P_n)$ of the path P_n is strongly multiplicative.

Proof: Let P_n be the path with n-vertices. Let $G = D_2(P_n)$ be the Shadow Graph of the path P_n.

Let $v'_1, v'_2, \ldots, v'_n$ be the vertices of the first copy P'_n and $v_1'', v_2'', \ldots, v_n''$ be the vertices of the second copy P_n'' of the path P_n. We note that $|V(G)| = 2n$ and $|E(G)| = 2n - 4$.

We define strongly multiplicative labeling $f : V(G) \rightarrow \{1,2,3,\ldots,2n\}$ as follows.

$f(v_i') = 2i; \quad 1 \leq i \leq n;$

$f(v_i'') = 2i-1; \quad 1 \leq i \leq n.$

The labeling pattern defined above satisfies the vertex conditions and edge conditions of strongly multiplicative labeling. That is, The Shadow Graph $D_2(P_n)$ of the path P_n is strongly multiplicative.

Illustration 2.6 : The Shadow Graph $D_2(P_7)$ and its strongly multiplicative labeling is shown in *Figure 3*

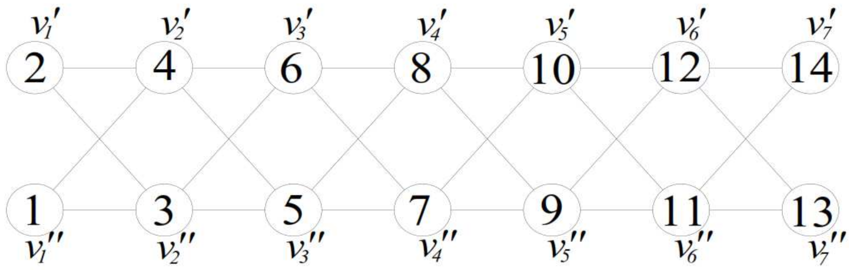

Figure 3: Strongly Multiplicative Labeling of Shadow Graph $D_2(P_7)$

Theorem 2.7: The Triangular Snakes TS_n are strongly multiplicative.

Proof: Let $G = TS_n$ be the Triangular Snake obtained from path P_n. Let $v_1, v_2,\ldots,v_n$ be the path vertices. We note that $|V(G)| = 2n -1$ and $|E(G)| = 3n -3$.

We define strongly multiplicative labeling $f : V(G) \rightarrow \{1,2,3,\ldots,2n-1\}$ as follows.

$f(v_i) = 2i -1; \quad 1 \leq i \leq n;$

$f(v_i') = 2i; \quad 1 \leq i \leq n-1.$

The labeling pattern defined above satisfies the vertex conditions and edge conditions of strongly multiplicative labeling. That is, The Triangular Snakes TS_n are strongly multiplicative.

Illustration 2.8: The Triangular Snake TS_5 and its strongly multiplicative labeling is shown in *Figure 4.*

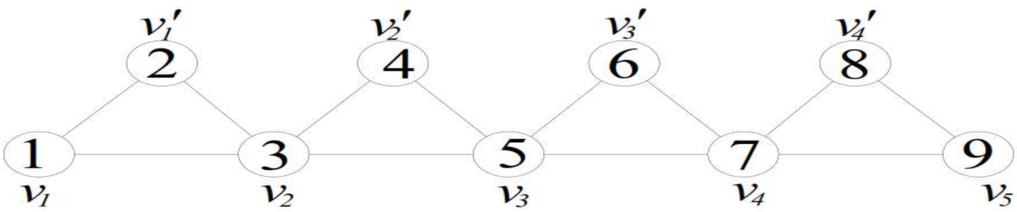

Figure 4: Strongly Multiplicative Labeling of Triangular Snake TS_5

CONCLUSIONS REMARKS

Here we have derived four results related to the strongly multiplicative labeling. To derive similar results related to other graph families is an open problem.

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